Gymnázium
Olomouc – Hejčín

# REQUIREMENTS FOR THE MATURITA

# EXAMINATION IN MATHEMATICS

# I. REQUIREMENTS FOR THE MATURITA EXAMINATION

## Logic and Proofs

**The student should be able to:**

- ✓ **express a sentence in ordinary English in symbols**
- ✓ **explain the mathematical meaning of words at most, at least, just one, each and none**
- ✓ **explain the term "statement"**
- ✓ **construct and use truth tables**
- ✓ **define and use logical operations and quantifiers, simplification of compound statements**
- ✓ **explain and construct direct and indirect proof, proof by induction.**

## Sets and Number Sets

**The student should be able to:**

- ✓ **explain the term "set" and list the ways of defining sets**
- ✓ **use set notation (including symbols for union and intersection) to define intervals**
- ✓ **recognise types of intervals and express intervals by means of inequalities**
- ✓ **explain the basic set operations and use Venn' s diagrams**
- ✓ **explain the difference between a digit and a number**
- ✓ **define the different number sets (Natural, Integer, Rational, Irrational and Real numbers) and appreciate how they fit on a number line**
- ✓ **explain the relationships between these number sets (eg.Natural numbers are a subset of Integers)**
- ✓ **define a binary operation**
- ✓ **list four basic binary operations**
- ✓ **determine if a given binary operation is associative, distributive and commutative, what its identity element is, and how the inverse of an element may be found**
- ✓ **explain the statement a is divisible by b and express it in a symbolic language**
- ✓ **explain the term "prime number"**
- ✓ **use the rules for divisibility and write proofs**
- ✓ **find the modulus of a real number, and state its geometrical significance.**
- ✓ **explain the basic properties of modulus of a real number**

## Algebraic Expressions

**The student should be able to:**

- ✓ **define a polynomial and perform four basic operations on polynomials**
- ✓ **define the domain of an algebraic expression**
- ✓ **find the value of a polynomial for a given value of x**
- ✓ **expand brackets**
- ✓ **expand the expressions $(a + b)^n$ and $(a - b)^n$ for n = 2, 3**
- ✓ **factorise**
- ✓ **factorise $a^n - b^n$ for n = 2, 3**
- ✓ **factorise $x^2 + px + q$, $ax^2+bx+c$ ( Viete's Formulae)**

- ✓ **collect like terms**
- ✓ **write an algebraic expression as a single fraction**
- ✓ **simplify algebraic expressions using any of the above methods**
- ✓ **change the subject of a formula**
- ✓ **simplify algebraic expressions using laws of indices and laws of log**
- ✓ **manipulate algebraic expressions involving a modulus**
- ✓ **explain the basic differences between a polynomial and an equation**

## Indices and Logarithms

**The student should be able to:**

- ✓ **explain the meaning of a zero index, natural, rational indices and negative indices**
- ✓ **simplify, evaluate algebraic and numerical expressions using the laws of indices and state conditions under which expressions are valid**
- ✓ **define $a^n$ for $n \in N$**
- ✓ **define the n-th root of a non-negative number a and state its basic properties( simplifying surds eg. $\sqrt{ab} = \sqrt{a} * \sqrt{b}$ )**
- ✓ **rationalise fractions involving surds**
- ✓ **convert expressions from index form to surd form and vice versa**
- ✓ **define a logarithm and the domain of a logarithmic function**
- ✓ **explain the basic properties of logarithms**
- ✓ **simplify algebraic and numerical expressions using the laws of logarithms.**

## Complex Numbers

**The student should be able to:**

- ✓ **define i as the square root of -1**
- ✓ **solve quadratic equations with complex roots**
- ✓ **define the real and imaginary parts of a complex number**
- ✓ **explain what is meant by equal complex numbers**
- ✓ **explain what is meant by a complex conjugate number and recognise the notation**
- ✓ **add, subtract, multiply, divide complex numbers in Cartesian form**
- ✓ **represent complex numbers on Gaussian plane (an Argand diagram)**
- ✓ **convert complex numbers from Cartesian form to a modulus-argument form and vice versa**
- ✓ **multiply and divide complex numbers in a modulus-argument form**
- ✓ **find the roots of a complex number**
- ✓ **use de Moivre's Theorem for integer exponents**
- ✓ **solve equations with complex numbers($z^n = a$).**
- ✓ **represent equations and inequalities including complex numbers on Gaussian plane (an Argand diagram)**

## Functions and Graphs

**The student should be able to:**

- ✓ **define a function and list different ways of describing functions**
- ✓ **describe the relationship between functions, mappings, binary relations and Cartesian products**
- ✓ **use function notation correctly**
- ✓ **find the domain and range of a function**
- ✓ **classify functions(odd, even, monotonic, one to one, many to one, periodic, asymptotic, having a maximum and a minimum)**
- ✓ **find the points of intersection of f(x) with the axes**
- ✓ **define and explain the inverse of a function and its properties**
- ✓ **determine if a function has an inverse**
- ✓ **find inverse functions algebraically and geometrically.**
- ✓ **combine functions to make a composite function**
- ✓ **recognise the graphs of basic functions (f(x)=c, f(x)=$x^n$,where n∈N,Z)**
- ✓ **sketch the graphs of the above basic functions**
- ✓ **recognise and sketch graphs of simple transformations (f(x) + a, f(x + a), af(x), f(ax), -f(x), f(-x)) and combinations of these**
- ✓ **define and explain the continuity of f(x) at a given point and interval, basic properties of continuity**
- ✓ **define and explain the limit of f(x) and its properties for** $x \to c, x \to \pm\infty$
- ✓ **find the limit of a function**

## Constant And Linear Functions, Equations and Inequalities

**The student should be able to:**

- ✓ **define a constant and a linear function and describe their properties**
- ✓ **recognise a constant and a linear function from its equation and from its graph**
- ✓ **draw the graph of a constant and linear function, given their equations and explain geometrical meaning of a and b in y= ax +b**
- ✓ **solve linear equations with one unknown**
- ✓ **solve systems of linear equations simultaneously, with up to three unknown**
- ✓ **solve linear inequalities both algebraically and graphically**
- ✓ **solve problems which lead to linear equations.**
- ✓ **solve parametric linear equations and their systems graphically and numerically**

## Quadratic Functions, Equations and Inequalities

**The student should be able to:**

- ✓ **define a quadratic function and describe its properties**
- ✓ **recognise a quadratic function from its equation and from its graph**
- ✓ **draw the graph of a quadratic function, given its equation(3 methods)**
- ✓ **explain geometrical meaning of a and c in y= $ax^2$+ bx + c**

- ✓ **solve a quadratic equation with one unknown by factorising, completing the square, using the formula or graphically**
- ✓ **use the discriminant to determine the nature of roots of a quadratic equation**
- ✓ **use Viete's formulae**
- ✓ **solve systems of quadratic and linear equations simultaneously(graphical and numerical solution)**
- ✓ **solve quadratic inequalities both algebraically and graphically**
- ✓ **solve problems leading to quadratic equations.**
- ✓ **solve parametric quadratic equations and their systems graphically and numerically**

## Quotient Functions, Equations and Inequalities

**The student should be able to:**

- ✓ **define a quotient function and explain its properties**
- ✓ **recognise a quotient function from its equation and from its graph**
- ✓ **explain relation between indirect proportionality and this function**
- ✓ **draw the graph of a quotient function**
- ✓ **solve quotient equations and inequalities numerically and graphically**
- ✓ **solve simultaneous equations containing this type of equations**
- ✓ **solve problems leading to quotient equations**

## Modulus Functions, Equations and Inequalities

**The student should be able to:**

- ✓ **define a modulus function and describe its properties**
- ✓ **recognise a modulus function from its equation and from its graph**
- ✓ **draw the graph of a given modulus function ( including functions containing two moduli)**
- ✓ **solve equations involving linear or quadratic modulus functions**
- ✓ **solve inequalities involving linear or quadratic modulus functions**

## Exponential and Logarithmic Functions and Equations

**The student should be able to:**

- ✓ **define an exponential function, including $y = e^x$**
- ✓ **explain its basic properties**
- ✓ **recognise an exponential function from its graph**
- ✓ **draw the graph of a given exponential function**
- ✓ **solve different types of exponential equations**
- ✓ **define a logarithmic function and describe its properties**
- ✓ **define $y = \ln x$ as the inverse of $y = e^x$**
- ✓ **recognise a logarithmic function from its graph**
- ✓ **draw the graph of a given logarithmic function**
- ✓ **solve different types of logarithmic equations**
- ✓ **solve simultaneous equations involving exponential and logarithmic equations**

## Trigonometric Functions, Expressions and Equations

**The student should be able to:**

- ✓ **define and use measurement in degrees and radian measure**
- ✓ **define the basic trigonometric ratios using the unit circle (sin x, cos x, tan x, cotan x)**
- ✓ **describe basic properties of trigonometric functions**
- ✓ **find the values of all basic trigonometric functions for $0^O$, $30^O$, $45^O$, $60^O$ and $90^O$ and their multiples for $k \in Z$**
- ✓ **recognise and sketch the graphs of y = sin x, y = cos x, y = tan x, y = cotan x, y = sec x, y = cosec x**
- ✓ **sketch the graphs of simple transformations of trigonometric functions**
- ✓ **define the Pythagorean identities, the addition formulae, the double angle identities, the factor formulae, the half angle formulae realise that tan x = sinx/cosx etc.**
- ✓ **select and use appropriate trig identities to simplify trigonometric expressions and to prove that a given identity is true**
- ✓ **select and use appropriate trigonometric identities to solve trigonometric equations**
- ✓ **write solutions to a trigonometric equation both within a specified range and in general**
- ✓ **find limits of simple trigonometric functions eg.** $\lim_{x\to 0} \frac{\sin x}{x}$, $\lim_{x\to 0} \frac{\tan x}{x}$ .

## Sequences and Series

**The student should be able to:**

- ✓ **define a sequence and a series**
- ✓ **describe differences between a function and a sequence**
- ✓ **list different ways of describing sequences**
- ✓ **recognise the pattern of a sequence and express the nth term algebraically both independently and as a recurrence formula**
- ✓ **generate consecutive terms of a sequence, given its recurrence formula**
- ✓ **generate consecutive terms of a sequence, given an independent algebraic expression for the nth term**
- ✓ **describe basic properties of a given sequence ( eg. graph, monotonic sequence, bounded sequences)**
- ✓ **define an AP and explain its basic properties**
- ✓ **recognise an arithmetic progression and find its common difference**
- ✓ **solve problems using the formulae for the nth term and the sum of an A.P.**
- ✓ **define a GP and explain its basic properties**
- ✓ **recognise a geometric progression and find its common ratio**
- ✓ **solve problems using the formulae for the nth term and the sum of a G.P. including interest rates**
- ✓ **calculate the sum to infinity of a G.P.**
- ✓ **define convergence and divergence of series**
- ✓ **explain how to use the formula for the sum of infinite geometrical series for different values of q**
- ✓ **solve problems based on properties of an AP, a GP and a geo series**

- ✓ **define and explain the limit of a sequence**
- ✓ **describe properties of limits**
- ✓ **find the limit of a convergent sequence.**
- ✓ **use sigma notation for series**

## Differentiation

**The student should be able to:**

- ✓ **explain what is meant by the derived function/derivative:**
  - ♦ **its definition from the first principle as a limiting value**
  - ♦ **its geometrical meaning as the gradient function**
  - ♦ **its physical meaning as a rate of change**
- ✓ **recognise and use the different forms of notation for differentiation**
- ✓ **differentiate basic functions and prove the results from the first principle**
- ✓ **describe the relationship between continuity and the first derivative of f(x) at a given point**
- ✓ **find the second derivative of a function and use it to identify stationary points**
- ✓ **differentiate products and quotients**
- ✓ **use differentiation to find the equation of a tangent and normal to a curve**
- ✓ **use the gradient function to determine if a given function is increasing or decreasing over a given interval**
- ✓ **find and identify stationary points as maxima/minima/points of inflexion (necessary and sufficient conditions)**
- ✓ **apply differentiation to problems in velocity and acceleration**
- ✓ **find and identify points of inflexion (necessary and sufficient conditions) and intervals in which a function is concave up/down**
- ✓ **use the above in sketching curves and in solving problems about optimisation**
- ✓ **use the chain rule in solving practical problems**
- ✓ **differentiate implicit functions**

## Curve Sketching

**The student should be able to:**

- ✓ **sketch the graphs of basic functions and simple transformations of these eg.** $f(x)=(x-2)^2+4$ **, labelling clearly any points of intersection with the axes**
- ✓ **sketch curves of more complicated functions by:**
  - ♦ **finding the domain and the range of a given function**
  - ♦ **investigating the basic properties of f(x)(eg.odd, even, periodicity, continuity)**
  - ♦ **investigating behavior of f(x) at points of discontinuity**
  - ♦ **finding points of intersection with the axes**
  - ♦ **finding stationary points and determining their nature**
  - ♦ **finding intervals in which f(x) is increasing(decreasing)**
  - ♦ **finding points of inflexion**
  - ♦ **finding intervals in which f(x) is concave up/down/**
  - ♦ **investigating asymptotes without gradient and asymptotes with gradient**
- ✓ **label sketches of curves correctly.**

## Integration

**The student should be able to:**

- ✓ **recognise integration as the reverse process of differentiation**
- ✓ **use Leibnitz notation**
- ✓ **explain primitive/antiderivative/ of f(x)**
- ✓ **explain properties of non-definite intergral**
- ✓ **recognise integration as a process of summation**
- ✓ **integrate a given function by means of formulae**
- ✓ **integrate a given function by parts and by substitution**
- ✓ **evaluate definite integral**
- ✓ **use integration to find the area under the curve and the volume of the solids**

## Combinatorics

**The student should be able to:**

- ✓ **use factorial notation**
- ✓ **solve problems based on combinations without/with repetition**
- ✓ **solve problems based on permutations without/with repetition**
- ✓ **describe the difference between a permutation and a combination**
- ✓ **use the notation**
- ✓ **explain basic properties of binomial coefficients**
- ✓ **solve more difficult problems involving permutations and combinations (with and without repetition of objects)**
- ✓ **recognise the connection between Pascal's Triangle and the Binomial coefficients**
- ✓ **state the Binomial Theorem**
- ✓ **use the Binomial Theorem to expand $(a+bx)^n$ for positive integer n and solve other exercises concerning it**
- ✓ **solve equations and simplify expressions containing binomial coefficients**

## Probability

**The student should be able to:**

- ✓ **explain the mathematical meaning of an event**
- ✓ **define the probability of an event taking place**
- ✓ **find the probability of an event based on random selection**
- ✓ **recognise mutually exclusive events A, B and use the addition rule to calculate the probability of (A or B)**
- ✓ **recognise independent events C, D and use the multiplication rule to calculate the probability of (C and D)**
- ✓ **draw and use space diagrams and tree diagrams**
- ✓ **recognise dependent events and calculate the associated conditional probabilities**
- ✓ **solve problems based on binomial probabilities**

## Statistics

**The student should be able to:**

- ✓ **explain the aims of statistics and their applications**
- ✓ **organise collected data in frequency tables**
- ✓ **calculate the mean, median, mode, standard deviation and quartiles of a set of data**
- ✓ **represent data graphically using bar charts, pie charts, histograms, frequency polygons, cumulative frequency graphs**
- ✓ **interpret and compare statistical data**

## Constructive Geometry in the Plane

**The student should be able to:**

- ✓ **explain what is meant by a point, a line, a half-line a line segment and a plane**
- ✓ **use the notation for the above**
- ✓ **find the locus of a point by means of construction**
- ✓ **express geometric ideas by means of symbolic language**
- ✓ **define an angle and classify angles**
- ✓ **define a circle and the parts of a circle**
- ✓ **explain basic properties of circles**
- ✓ **solve problems leading to construction of cirles**
- ✓ **calculate angles using the circle angle theorems**
- ✓ **calculate areas of parts of a circle**
- ✓ **define a triangle and a quadrilateral, their basic elements and their basic properties**
- ✓ **classify triangles and quadrilaterals**
- ✓ **construct a triangle given by three elements of the triangle, and produce an analysis, a description and a discussion of the construction**
- ✓ **prove if two triangles are congruent or similar**
- ✓ **use the Pythagorean and Euclidean theorems for triangles**
- ✓ **define the different types of transformation and their basic properties (translation, reflection, rotation and dilatation)**
- ✓ **use the above transformations in construction problems**
- ✓ **calculate the area and circumference of a circle.**
- ✓ **calculate the areas and perimeters of polygons, segments and sectors of circles**

## Trigonometry

**The student should be able to:**

- ✓ **define the basic trigonometric ratios**
- ✓ **find the missing sides/angles in a right-angled triangle, using the trig ratios**
- ✓ **define and write the proof of the sine and cosine rules**
- ✓ **find the missing sides/angles in any triangle, using the sine and cosine rules**
- ✓ **solve topographic problems leading to the sine and cosine rules**
- ✓ **solve three-dimensional trig problems.**
- ✓ **Solve topographical tasks concerninng bearngs, the angles of depression and elevation**

## Constructive Geometry in Space

**The student should be able to:**

- ✓ **describe the different configurations in space of:**
  - ♦ **a point and a line**
  - ♦ **a point and a plane**
  - ♦ **two lines**
  - ♦ **a line and a plane**
  - ♦ **three planes**
- ✓ **explain theorems concerning different configurations points, lines and planes in space**
- ✓ **list the conditions for lines and planes to be parallel or perpendicular**
- ✓ **define the angle between 2 intersecting lines, a line and a plane, 2 planes and skew lines**
- ✓ **calculate the angle between:**
  - ♦ **two intersecting lines**
  - ♦ **a line and a plane**
  - ♦ **two planes**
  - ♦ **two skew lines**
- ✓ **construct and calculate the perpendicular distance of a point from a line and the perpendicular distance of a point from a plane**
- ✓ **construct and calculate the perpendicular distance of a line /a plane/ from a parallel plane**
- ✓ **define a solid and explain its basic properties**
- ✓ **classify solids and describe their basic properties**
- ✓ **construct solids using the parallel projection**
- ✓ **construct sections through solids and explain basic principles of used mappings**
- ✓ **explain Cavallieri´s principle**
- ✓ **calculate the volumes and surface areas of solids and their parts**
- ✓ **solve three-dimensional problems using trigonometry or vectors.**

## Vector Geometry

**The student should be able to:**

- ✓ **explain what is meant by a vector**
- ✓ **add, subtract and find scalar multiples of vectors in two-dimensions (numerically and graphically) and in three-dimensions (numerically)**
- ✓ **find the magnitude of a vector**
- ✓ **recognise and write vector equations of lines, half-lines and line segments in two- and three- dimensions**
- ✓ **determine if two given vectors are intersecting, parallel or skew**
- ✓ **define the scalar product and list its main properties**
- ✓ **use the scalar product in solving problems**
- ✓ **explain difference between the direction vector and the normal vector of a line in the plane**
- ✓ **understand the significance of normal vectors**
- ✓ **calculate angles in two- or three-dimensional situations involving vectors**

- ✓ **define the vector product and list ist geometrical properties**
- ✓ **use the vector product in solving problems.**

## Coordinate Geometry in the Plane – the Straight Line

**The student should be able to:**

- ✓ **define 3 types of equations of a line**
- ✓ **change from one form of the equation of a line to the other**
- ✓ **find the mid-point and length of a line, given the coordinates of its end-points**
- ✓ **find the gradient of a line, given the coordinates of two points on the line**
- ✓ **recognise the equation of a straight line, y = kx + q, and explain the geometrical significance of k and q**
- ✓ **determine if two given lines are parallel or intersecting especially if they are perpendicular**
- ✓ **write the equation of a line given the coordinates of two points on the line or given its gradient and the coordinates of one point on the line**
- ✓ **find the equations of lines going through a given point and having a certain angle to a given line**
- ✓ **find the coordinates of the point of intersection of two lines**
- ✓ **find the angle between two lines**
- ✓ **find the perpendicular distance of a given point from a given line**
- ✓ **find the perpendicular distance between 2 paralllel lines**
- ✓ **find the locus of a point using co-ordinate geometry.**

## The Circle

**The student should be able to:**

- ✓ **define a circle as the locus of points and find its equation**
- ✓ **define a circle as the locus of points and derive its equation from its definition**
- ✓ **recognise the equation of a circle in the form $(x-m)^2+(y-n)^2=r^2$ and in the form $x^2+y^2+2Ax+2By+C=0$, and be able to convert one form in to the other**
- ✓ **write the equation of a circle , given the coordinates of its centre and the length of its radius**
- ✓ **find the centre and radius of a circle, given its equation**
- ✓ **find the equation of a circle/ the coordinates of the centre and radius given by three points**
- ✓ **find the equation of a tangent to a circle:**
  - ♦ **at a given point**
  - ♦ **parallel to a given line**
  - ♦ **perpendicular to a given line**
- ✓ **find the length of a tangent to a circle from a given point**
- ✓ **find the angle between tangents**

- ✓ **describe configurations of a circle and a line or two circles**
- ✓ **determine if two circles intersect or touch internally or externally, given their equations**
- ✓ **solve a variety of circle problems with the aid of basic geometric facts: eg. the perpendicular bisector of a chord goes through the centre of the circle.**
- ✓ **define a sphere and describe the basic properties of a sphere, the plane of tangency**
- ✓ **solve problems involving spheres**

## Conic Sections

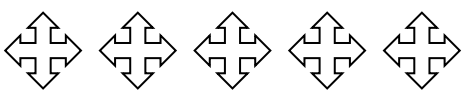
**The student should be able to:**

- ✓ **define the parabola, the ellipse and the hyperbola as the locus of a point**
- ✓ **describe the construction of the parabola, the hyperbola and the ellipse**
- ✓ **give the equations and describe the basic properties of the parabola, ellipse and hyperbola**
- ✓ **describe configurations of a line with each one of: the parabola, the hyperbola and the ellipse**
- ✓ **find the tangent to these curves at a given point of tangency and at a point**
- ✓ **find the angle between tangents**
- ✓ **find the equation of a tangent to a given curve being parallel/perpendicular to a given line**

## Co-ordinate Geometry in Space

**The student should be able to:**

- ✓ **define the vector equation of a line, a half-line and a lines segment**
- ✓ **explain a collinear/ non-collinear point**
- ✓ **explain the basic configuration of lines in space**
- ✓ **find the vector equation of a line going through a given point parallel/perpendicular to a given line**
- ✓ **the angle between 2 lines in space**
- ✓ **define the vector equation and the Cartesian equation of a plane, the half-plane**
- ✓ **explain the basic configrations of a point, a line and a plane and 2 planes**
- ✓ **find the angle between a line, a plane and 2 planes**
- ✓ **find a perpendicular plane to given 2 planes going through a given point**
- ✓ **find the perpendicular distance of a point from a line/a plane**
- ✓ **find the perpendicular distance of a parallel line/plane from a plane**



## II. THE DESCRIPTION OF THE MATURITA EXAMINATION

**In Mathematics the Maturita Examination consists of two parts :**

- ***WRITTEN EXAMINATION*** **(April). There are two parts.**

  **In part A students are asked to solve 15 simple examples in the form of a multiple choice test. They can use only non-graphical calculators. The time limit is 60 minutes.**

  **In part B students are asked to choose and solve four out out of six complex examples. Tables and non-graphical calculators can be used. Time limit is 180 minutes.**

  **The maximum mark is 60 points.**

- ***ORAL EXAMINATION*** **(May). Students are asked to choose a question dealing with a particular area of Mathematics. Students should demonstrate complex understanding of problems (based mainly on the theoretical principles proved on the particular examples). Time limit is 15 minutes.**

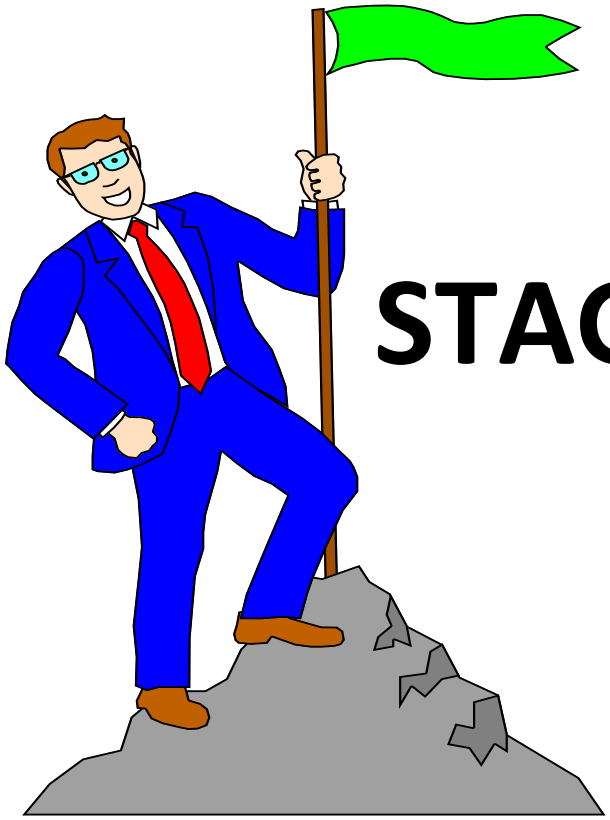
  **The maximum assessment is 40 points.**

**Prostějov 9.9.2019 8:25:19.**

**NOTES:**

Gymnázium
Olomouc – Hejčín

# STAGE 1



# STAGE 2